meters. The following results may then be derived for the incident shock, using the notation given in Fig. 1:

$$U_{10} = 2(W_{10}^2 - 1)/(\gamma+1)W_{10} \quad (12)$$

$$p_1/p_0 = [2\gamma W_{10}^2 - \gamma + 1]/(\gamma+1) \quad (13)$$

$$A_{10}^2 = [2\gamma W_{10}^2 - \gamma + 1][(\gamma-1)W_{10}^2 + 2]/(\gamma+1)^2 W_{10}^2 \quad (14)$$

where

$$W_{10} = G_1/a_0, \quad U_{10} = u_1/a_0, \quad A_{10} = a_1/a_0, \quad W_{20} = G_2/a_0$$

The pressure ratio across the reflected shock can be obtained as

$$p_2/p_1 = [2\gamma(U_{10} + W_{20})^2 - (\gamma-1)A_{10}^2]/(\gamma+1)A_{10}^2 \quad (15)$$

and the Mach number of the gas behind the reflected shock is

$$M_2 = \left[\frac{(\gamma-1)(U_{10}+W_{20})^2 + 2A_{10}^2}{2\gamma(U_{10}+W_{20})^2 - (\gamma-1)A_{10}^2}\right]^{1/2} - \frac{W_{20}}{A_{20}} \quad (16)$$

where

$$A_{20}^2 = \frac{[2\gamma(U_{10}+W_{20})^2 - (\gamma-1)A_{10}^2][(\gamma-1)(U_{10}+W_{20})^2 + 2A_{10}^2]}{(\gamma+1)^2(U_{10}+W_{20})^2} \quad (17)$$

Finally, from considerations of mass conservation across the reflected shock, an expression involving the quantity Q of Eq. (2) can be obtained:

$$\frac{\varepsilon Q}{\rho_0 a_0} = \left[\frac{W_{10}}{W_{10} - U_{10}}\right]\left\{U_{10} + W_{20}\left[\frac{A_{10}^2 - (U_{10}+W_{20})^2}{A_{10}^2 + \frac{1}{2}(\gamma-1)(U_{10}+W_{20})^2}\right]\right\} \quad (18)$$

When the initial state of the gas and the characteristics of the permeable material are specified, the above equations can be solved by a simple numerical procedure to yield W_{20} as a function of W_{10}. The computation makes use of the fact that $p_5 = p_2$, and allowance is made for the possibility of choking at state 4.

Experiments

The experiments were carried out in a conventional diaphragm shock tube of 3-in. × 3-in. cross section, using air as the working medium. Three different types of permeable material were used, classified as Foametal, Feltmetal and granular. Foametal consists of a latticework of metallic fibres such that there are no free fibre ends within the material, whereas Feltmetal consists of short sintered fibres with free ends of fibres lying within the material. The properties of these materials were taken from earlier experiments,[4] and are listed in Table 1. The granular material consisted of aluminum oxide grains of uniform size held together with a ceramic bond. The values of ε and k were known from earlier work, and a value for c was deduced from experimental results on the flow through beds of polyethylene particles. The surface of the permeable plugs which faced the incident shock was machined flat, so that the resulting surface consisted of a flat plane with a large number of pores penetrating into the material.

Results

Initial experiments were performed using a smooth flat impermeable wall as a reflecting surface, with a range of incident shock Mach numbers chosen to correspond to the range used in the experiments with the permeable plugs. All incident and reflected shock speeds were obtained by timing over distances of a few inches. The speeds were checked at several distances from the plug and found to be closely constant. The measured reflected shock speeds are compared with calculated values in Fig. 2, where the calculated curve is that for air with the vibrational energy included. Figure 2 also compares the experimentally determined reflected shock speeds from the permeable plugs with the computed values. It is seen that the analytical model agrees fairly well with the experiments, except for the Foametal plug. There are two possible reasons for this discrepancy: 1) the material is exceedingly porous so that a transmitted shock may have occurred and 2) the material tended to exhibit some distortion after several shock reflections, so that the plug had to be changed periodically. However, it would appear from the other results that the analytical model gives a reasonable prediction of the reflected shock speeds for permeable materials which have porosity values such that transmitted shocks do not occur.

Table 1 Properties of permeable plugs

Material	L, in.	ε	k, in.$^2 \times 10^6$	c
Foametal	1.0	0.95	15.0	0.075
Feltmetal	0.25	0.80	0.8	0.132
Granular I	1.0	0.35	1.0	0.26
Granular II	1.0	0.35	2.5	0.26

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Unified Area Rule for Hypersonic and Supersonic Wing-Bodies

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FOR low supersonic flow there exists the well-known area rule by which the effect of wing-body interference on over-all forces can be obtained without a knowledge of the details of the local flowfield for a wide class of practically interesting configurations. In the hypersonic range an analogous theorem was given by Ladyzhenskii[1] for the wave drag of blunted-nose bodies at zero incidence. The lifting case was recently studied by Malmuth[2] who derived a new area rule for the change in the aerodynamic efficiency L/D of a hypersonic delta wing due to the addition on its compression side of a conical body of arbitrary shape. However, his analysis is restricted to hypersonic flow past wing-body configurations at small incidence. Furthermore, the conically subsonic flow region, on which the conical body is added, is assumed a small portion of the wing. There is obviously a Mach number range for which neither the supersonic area rule nor Malmuth's hypersonic area rule can be applied.

On the other hand, a unified theory for flow past delta wings was given by the author[3] which is valid for both hypersonic and supersonic flow past delta wings of any sweep angle at any incidence, provided the shock wave is attached to the leading edges. It gives almost identical results compared with large scale numerical solutions. The purpose of this paper is to show, by combining the methods of Refs. 2 and 3, that all the restrictions

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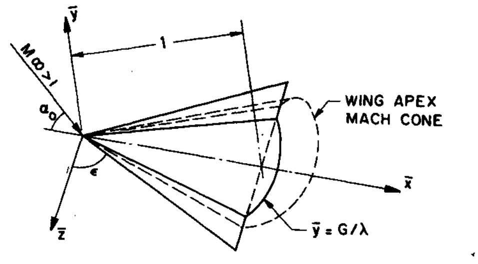

Fig. 1 Configuration showing notation.

mentioned above in Malmuth's paper can indeed be removed, and the area rule governing the total force applies for both hypersonic and supersonic flow with attached shock wave past delta wing with the addition on its compression side of a conical body of arbitrary shape.

Consider the flow over the compression side of a delta wing-body configuration at an incidence to a hypersonic or supersonic freestream (see Fig. 1). The same notation as Ref. 3 will be used and the combination of the freestream Mach number, the angle of incidence and the sweep-back angle is assumed to be such that the bow shock is attached to the leading edges. In the outer regions near the leading edges the flow is uniform and conically supersonic, and can be calculated exactly. On the other hand, in the inner region, which borders the outer regions at the wing apex Mach cone, the flow is nonuniform and conically subsonic, and is difficult to calculate exactly. However, it is shown in Ref. 3 that the flow in the inner region differs only slightly from the corresponding two-dimensional flow past a flat plate obtained by reducing the sweep angle to zero. This flat plate flow will be called reference flow and referred to with a subscript o. Thus u_o, p_o, ρ_o and M_o are, respectively, the velocity, pressure, density and Mach number of the reference flow.

Let δ be a small parameter characterizing the departure of the real flow in the inner region from the reference flow, and write the physical quantities as

$$\bar{u} = u_o(1+u), \quad \bar{v} = u_o v, \quad \bar{w} = u_o w$$
$$\bar{p} = p_o[1+(\gamma M_o^2/\lambda)p], \quad \bar{\rho} = \rho_o[1+(M_o^2/\lambda)\rho] \qquad (1)$$
$$\lambda = (M_o^2 - 1)^{1/2}$$

then $u = 0(\delta)$, etc. Conical coordinates η and ζ are introduced as below

$$\eta = \lambda\bar{y}/\bar{x}, \quad \zeta = \lambda\bar{z}/\bar{x} \qquad (2)$$

Let the equation of the additional symmetrical conical body on the compression side be

$$\eta = G(\zeta) \qquad (3)$$

and assume that G and $dG/d\zeta$ are $0(\delta)$. Furthermore, the addition of the conical body is limited to the wing apex Mach cone in the reference flow, i.e., $G = 0$ for $|\zeta| \geqq 1$. Under these conditions, the addition of the conical body will not affect the flow in the outer regions, and also the flow in the inner region will still differ from the reference flow by $0(\delta)$.

When terms $0(\delta^2)$ are neglected, the mixed boundary value problem for the perturbation pressure p in the inner region is[3]

$$(\eta^2-1)\frac{\partial^2 p}{\partial \eta^2} + 2\eta\zeta\frac{\partial^2 p}{\partial\eta\,\partial\zeta} + (\zeta^2-1)\frac{\partial^2 p}{\partial\zeta^2} + 2\eta\frac{\partial p}{\partial\eta} + 2\zeta\frac{\partial p}{\partial\zeta} = 0 \qquad (4a)$$

$$p = 0, \quad \text{at} \quad \eta^2+\zeta^2 = 1 \qquad (4b)$$

$$\frac{\partial p}{\partial\zeta} = 0, \quad \text{at} \quad \zeta = 0 \qquad (4c)$$

$$\frac{\partial p}{\partial\eta} = -\frac{\zeta^2}{\lambda}G''(\zeta), \quad \text{at} \quad \eta = 0 \qquad (4d)$$

$$\frac{\partial p}{\partial\eta} + \left[-\frac{A_o+H}{1-H^2}\zeta + \frac{HB_o}{(1-H^2)\zeta}\right]\frac{\partial p}{\partial\zeta} = 0, \quad \text{at} \quad \eta = H \qquad (4e)$$

$$\int_o^{(1-H^2)^{1/2}} \frac{dp}{\zeta} = w^*/B_o, \quad \text{at} \quad \eta = H \qquad (4f)$$

where A_o, B_o, H and w^* are constants given in Ref. 3 [see, respectively, Eqs. (22, 16 and 5)].

This problem is in exactly the same form as that of Ref. 2. Defining the spanwise integral of pressure by

$$P \equiv \int_o^{(1-\eta^2)^{1/2}} p\,d\zeta \qquad (5)$$

we have,

$$P''(\eta) = 0, \qquad P'(0) = -V/\lambda$$
$$P'(H) + [(A_0+H)/(1-H^2)]P(H) = -Hw^*/(1-H^2) \qquad (6)$$

The solution of (6) is

$$P = -(V/\lambda)\eta + [V(1+A_oH)/\lambda(A_o+H)] - w^*H/(A_o+H) \qquad (7)$$

where

$$V = 2\int_o^1 G(\zeta)\,d\zeta \qquad (8)$$

is proportional to the volume of the additional body.

With the spanwise integral of pressure given by Eq. (5), the total pressure force $\vec{F}$ on the wing-body is found as follows†

$$\vec{F} = \frac{p_o}{\lambda^2}\left[\vec{i}\,V - \vec{j}\left\{\lambda + \frac{\gamma M_o^2}{A_o+H}(V + A_oHV - w^*H)\right\}\right] + \vec{F}_1 \qquad (9)$$

where $\vec{i}$ and $\vec{j}$ are, respectively, the unit vectors in $\bar{x}$ and $\bar{y}$ direction, and $\vec{F}_1$ represents the force contribution from the outer regions of the compression surface, and from the expansion surface (this may not be negligible for moderate supersonic flow) of the wing-body configuration. Since $\vec{F}_1$ is not affected by the addition of the conical body within the wing apex Mach cone, we conclude from Eq. (9) that the total pressure force on a delta wing-body configuration with attached shock wave in hypersonic or supersonic flow depends linearly on the volume of the additional conical body, and is independent of its cross-sectional shape. This constitutes the extension of Malmuth's hypersonic area rule.

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† The base pressure is neglected in this analysis.

Laser Velocimeter Measurement of Reynolds Stress and Turbulence in Dilute Polymer Solutions

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